

Development of a PID Based Control for a Cantilever Beam Using Piezoelectric Transducers

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Abstract— Piezoelectric smart materials are widely used in modern day structures due to their effective controllability and easy integration into any complex shaped systems. This paper reports the development of a PID based strain feedback controller using ANSYS APDL code implemented in a smart cantilever beam with single/double piezo-patch configurations. Transient analysis is performed to demonstrate the effectiveness of collocated patches over single patch configuration. Parametric study on the piezo patch thickness and area revealed that the effect of self-capacitance (impedance) on the dynamic response was considerable, for including the same in the calculation of total impedance.

Keywords— 3D brick element; PID control; capacitance; open and short circuits; Strain feedback; harmonic analysis; ANSYS.

I. INTRODUCTION

Piezoelectric transducers have become increasingly popular in vibration control applications. They are used as sensors as well as actuators in structural vibration control systems due to their excellent actuation and sensing capabilities. The ability of piezoelectric materials to transform mechanical energy into electrical energy and *vice versa* was discovered over a century ago by Pierre and Jacques Curie. These French scientists discovered a class of materials that when pressured, generate electrical charge, and when placed inside an electric field, strain mechanically (fig. 1).

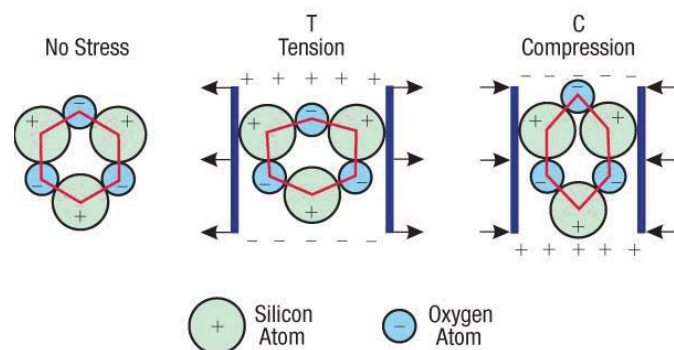


Fig (1) Behavior of Piezoelectric Transducer (Quartz) when a Stress is applied

The active control of structures is an important emerging field in which complex structures are effectively damped using

modern vibration control techniques. Piezo drives are becoming more and more popular since they can deliver extremely high accelerations which are very important in applications such as beam steering and optic stabilization. Piezoelectric shunt damping was analytically studied by C. H. Nguyen and S. J. Pietrzko in an asymmetrically patched cantilever beam to understand the behaviour of capacitance, resistance and inductance in dissipation of energy, harvested through vibration [9] [10]. Placement and dimension optimisation of piezo patch on a cantilever beam was studied analytically and published recently by J. Ducarne et al [3]. The behaviour of cantilever beam when subjected to proportional (P) control using single piezoelectric patch was analysed and experimentally studied by L. Malgaca and H. Karagulle [12] [13]. Akash et al. [11] studied the effect of single and collocated patch in determining the optimum control voltage for voltage clipping in active control by performing harmonic analysis on a smart cantilever beam.

In this work, piezoelectric transducers are used as sensors and actuators in vibration control. For this purpose, transducers are bonded to a flexible structure and utilized as either a sensors (*to monitor structural vibrations*), or as an actuators (*to add damping to the structure*). To develop model-based controllers capable of adding sufficient damping to a structure using piezoelectric actuators and sensors it is vital to have models that describe the dynamics of such systems with sufficient precision and accuracy.

II. FINITE ELEMENT MODELLING OF 3D CANTILEVER BEAM WITH PIEZOELECTRIC PATCH

To continue the analysis on the effect of piezoelectric patch coupled with a cantilever beam, it is required to make a combined model for the same. Initially a basic spring mass system is modelled in order to understand active vibration control procedure using FEM.

A. Modelling of 3 Degrees of Freedom Spring Mass System

A single-degree-of-freedom spring-mass system has one natural mode of oscillation. A three degree-of-freedom spring-mass system (*consisting of three identical masses connected between three identical springs*) has three distinct natural modes of oscillation. At first, the modal analysis of three

degrees of freedom system is performed to deduce natural frequency of the system. The vibration schematics can be obtained by conducting transient analysis. Ultimately, Proportional Integral Derivative (PID) controller is implemented to damp out the vibration. The spring mass system is developed in order to understand how the same may be introduced to a cantilever beam with piezoelectric patch when induced to PID damping. It is a basic model to understand the phenomenon of vibration in terms of finite element theories.

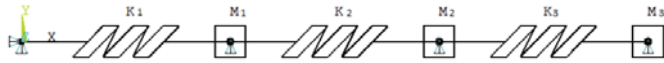


Fig (2) Three Degrees of Freedom Spring Mass System

Mass 21 (mass) and COMBIN 14 (spring) are the elements used to model the spring mass system in the ANSYS. Natural frequencies are obtained by performing modal analysis of three degrees of freedom system. Transient analysis is performed with the control loop to simulate PID based control. The equation governing this system is

$$m\ddot{x} + kx = F_e(x) \tag{1}$$

B. Modelling of 3D Cantilever Beam with Single Piezo Patch

Finite element modelling is considered to be one of the most effective way of computing and analyzing the mechanical parameters of a complex system. ANSYS offers simulated solutions in engineering that a design process requires. A three dimensional model of a cantilever beam with single piezoelectric patch is created initially so as to determine the effect of the same when it is asymmetrically coupled with the beam. Solid186 which is a 20 node element and has 3 degrees of freedom per node is used to model the structure. The material and geometrical properties of the smart beam is taken from [11-13]. The piezoelectric patch is modelled using Solid226 which is also a 20 node element having four degrees of freedom (U_x, U_y, U_z and $volt$) per node. Parametric study is conducted by varying the length and thickness of the piezo patch to deduce the effect of self-capacitance on the system dynamics [2]. When operated well below the resonant frequency, a piezo actuator behaves as a capacitor. The actuator displacement is proportional to stored charge. The capacitance of the actuator depends on the area and thickness of the ceramic, as well as on its dielectric properties. The equation governing the capacitance of piezoelectric patch is

$$C_p = \epsilon_0 \epsilon_{33} \frac{A_s}{h_s} \tag{2}$$

Where, $\epsilon_0 = 8.854 \times 10^{-12} \text{F/m}$ is the electric permittivity of vacuum, $\epsilon_{33} = 1468$ is normalized permittivity in the polarized direction of PIC 141. [9]

The material properties of the piezoelectric material are determined by the following constants.

Dielectric Matrix ϵ_{ik}

$$\begin{bmatrix} 15.03 & 0 & 0 \\ 0 & 15.03 & 0 \\ 0 & 0 & 13 \end{bmatrix} \times 10^{-9} \text{ F/m}$$

Stiffness Matrix C_{jh}

$$\begin{bmatrix} 12.6 & 7.95 & 8.41 & 0 & 0 & 0 \\ & 12.6 & 8.41 & 0 & 0 & 0 \\ & & 11.7 & 0 & 0 & 0 \\ & & & 2.33 & 0 & 0 \\ & & & & 2.3 & 0 \\ & & & & & 2.3 \end{bmatrix} \times 10^{10} \text{ Pa}$$

Piezoelectric stress matrix e_{ij}

$$\begin{bmatrix} 0 & 0 & -6.5 \\ 0 & 0 & -6.5 \\ 0 & 0 & 23.3 \\ 0 & 0 & 0 \\ 0 & 17 & 0 \\ 17 & 0 & 0 \end{bmatrix} \text{ C/m}^2$$

The piezoelectric consecutive equations can be expressed in the matrix form as

$$[T] = [c]\{S\} - [e]^T \{E\} \tag{3}$$

$$[D] = [e]\{S\} + [\epsilon]\{E\} \tag{4}$$

Where, $[T]$ is the strain matrix, $[c]$ is the matrix of compliance coefficient, $\{S\}$ is the stress matrix, $[e]$ is the matrix of piezoelectric strain constant, $\{E\}$ is the matrix of applied electric field, $[D]$ is the matrix of electric displacement and $[\epsilon]$ is the permittivity matrix.

Mechanical Strain can be given as

$$S^e = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial z} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} \tag{5}$$

Here u, v and w are the displacements in X, Y and Z directions respectively. The displacements u, v and w are

expressed as functions of nodal displacements u_i, v_i and w_i and the nodal shape functions N_i ($i=1, 2, 3, n$ where n is the total number of nodes for the element) and

$$u = \sum_{i=1}^n N_i u_i, v = \sum_{i=1}^n N_i v_i, w = \sum_{i=1}^n N_i w_i \quad (6)$$

The displacement vector $\{u^e\}$ and the electric field vector $\{E^e\}$ are given by

$$\{u^e\} = \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} \quad (7)$$

$$\{E^e\} = \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} = - \begin{Bmatrix} \frac{\partial \phi}{\partial x} \\ \frac{\partial \phi}{\partial y} \\ \frac{\partial \phi}{\partial z} \end{Bmatrix} \quad (8)$$

The nodal potentials and nodal shape functions are related as

$$\phi = \sum_{i=1}^n N_i \phi_i$$

Using this above equation becomes

$$\{E^e\} = - \begin{Bmatrix} \frac{\partial \sum_{i=1}^n N_i \phi_i}{\partial x} \\ \frac{\partial \sum_{i=1}^n N_i \phi_i}{\partial y} \\ \frac{\partial \sum_{i=1}^n N_i \phi_i}{\partial z} \end{Bmatrix} \quad (9)$$

Above equation can be written as

$$\{E^e\} = - \{B_\phi\} \{\phi^e\} \quad (10)$$

As the nodal shape functions are functions of the coordinates. Therefore,

$$\{B_\phi\} = \begin{Bmatrix} [N_x] \\ [N_y] \\ [N_z] \end{Bmatrix} \quad (11)$$

By equating the internal virtual work with the work done by the external mechanical forces and the external electrical charges, we get the elemental stiffness matrix for a piezoelectric finite element, as

$$\int_V \{\delta S^e\}^T \{T^e\} dV = \{\delta u^e\} \{F^e\} - \int_V \{\delta E^e\} \{D^e\} dV = \{\delta u^e\}^T \{Q^e\} \quad (12)$$

Substituting these equation to the primary constitutive equations (3) and (4)

$$\begin{aligned} \{T^e\} &= [c][B_u]\{u^e\} + [e]^T [B_\phi] \{\phi^e\} \\ \{D^e\} &= [e][B_u]\{u^e\} + [\varepsilon]^T [B_\phi] \{\phi^e\} \end{aligned} \quad (13)$$

We also have

$$\begin{aligned} \{\delta S^e\}^T &= \{\delta u^e\}^T [B_u]^T \\ \{\delta E^e\}^T &= -\{\delta \phi^e\} [B_\phi]^T \end{aligned} \quad (14)$$

Using equations (13) and (14) in equation (12), we get

$$\begin{aligned} \{\delta u^e\} \int_V [B_u]^T [c][B_u]\{u^e\} + [B_u]^T [e]^T [B_\phi] \{\phi^e\} dV &= \{\delta u^e\}^T \{F^e\} \\ \{\delta \phi^e\}^T \int_V [B_\phi]^T [e][B_u]\{u^e\} - [B_\phi]^T [\varepsilon][B_\phi] \{\phi^e\} dV &= \{\delta \phi^e\}^T \{Q^e\} \end{aligned} \quad (15)$$

Rearranging the above equations, the finite element equations for an element can be written as

$$\begin{Bmatrix} [K_{uu}^e] & [K_{u\phi}^e] \\ [K_{\phi u}^e] & [K_{\phi\phi}^e] \end{Bmatrix} \begin{Bmatrix} \{u^e\} \\ \{\phi^e\} \end{Bmatrix} = \begin{Bmatrix} \{F^e\} \\ \{Q^e\} \end{Bmatrix} \quad (16)$$

Where,

$$\begin{aligned} [K_{uu}^e] &= \int_V [B_u]^T [c][B_u] dV \\ [K_{u\phi}^e] &= [K_{\phi u}^e]^T = \int_V [B_u]^T [e]^T [B_\phi] dV \\ [K_{\phi\phi}^e] &= \int_V [B_\phi]^T [\varepsilon][B_\phi] dV \end{aligned}$$

For the entire mesh elemental equations are added to get the global finite element equations as

$$\begin{Bmatrix} [K_{uu}] & [K_{u\phi}] \\ [K_{\phi u}] & [K_{\phi\phi}] \end{Bmatrix} \begin{Bmatrix} \{u\} \\ \{\phi\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{Q\} \end{Bmatrix} \quad (17)$$

A comparison on cantilever beam with single patch and collocated patch are consecutively studied to understand the response of the beam when induced to PID based strain feedback control mechanism.

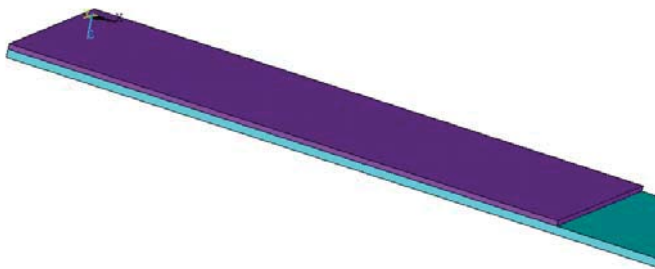


Fig (3) Cantilever Beam with Single Piezoelectric Patch

C. Modelling of 3D Cantilever Beam with Two Piezo Patch

To analyze the effect of symmetric piezo patch arrangement on a cantilever beam, it is required to make a combined model of a beam with two patch. From the literature review carried out it is understood that symmetric arrangement of piezo patch can be more effective in damping [3].

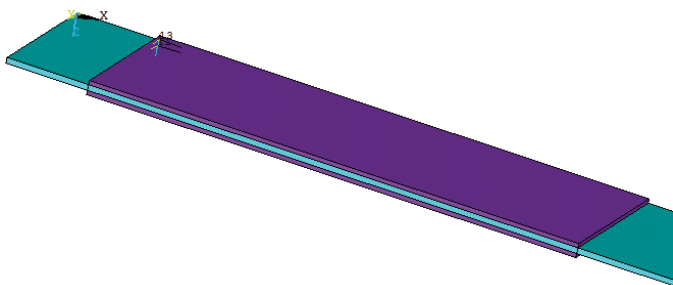


Fig (4) Cantilever Beam Coupled with Two Piezo Patch

III. OPEN AND SHORT CIRCUITRY

Open-circuit voltage is the difference of electrical potential between two terminals of a device when disconnected from any circuit. There is no external load connected. No external electric current flows between the terminals. For modelling the same, one end of the piezoelectric patch is open (*volt dofs are coupled*) and the other end is grounded (*0 volt*). When a bending happens on the beam, a potential difference is generated in the patch and charge gets stored in the top plate developing self-capacitance which modifies the system dynamics.

A short circuit is an electrical circuit that allows a current to travel along an unintended path, often where essentially no electrical impedance is encountered. A short circuit is said to be an abnormality between two nodes of an electrical circuit intended to be at different voltages. For modeling the same, top and bottom electrodes are applied with the same potential (*0 volts*). This method will help to understand the behavior of piezo patch where no self-capacitance is generated, when induced to bending. This study on open and short circuitry is essential to understand the effect self-capacitance in the system dynamics and the role of the same in the overall impedance.

IV. PID BASED VIBRATION CONTROL

A Proportional-Integral-Derivative (PID) controller is a generic control loop feedback mechanism widely used in

industrial control systems. The controller attempts to minimize the error by adjusting the process control outputs. The proportional, integral and derivative are the three different parameters in a PID controller which is precisely tuned to its optimum by conducting a series of analysis carried out by continuously applying a series of random values, to achieve the best damping for the system when affected by resonant frequencies. A transient analysis can be carried out to understand the behavior of a structure, when exposed to an impulsive force. The response of a system while increasing the proportional, integral and derivative parameters in controlled damping is listed in table 1.

Control Response	Rise Time	Overshoot	Settling Time	S-S Error
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	No Change

Table (1) Effects of Increasing a Parameter Independently

Modern industrial facilities no longer tune loops using the manual calculation methods shown above. Instead, PID tuning and loop optimization software are used to ensure consistent results. These software packages will gather the data, develop process models, and suggest optimal tuning. Some software packages can even develop tuning by gathering data from reference changes. In this work the PID based control for cantilever beam is simulated using APDL, by applying the values for P, I and D as 1, 1 and 0.00055

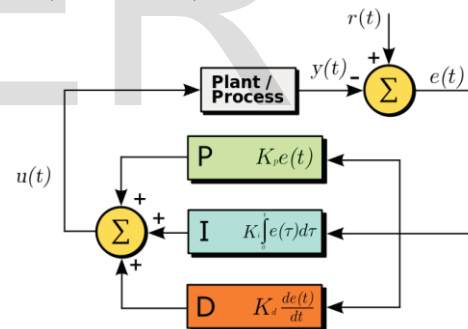


Fig (5) A Block Diagram of a PID controller in a feedback loop

A. Optimum Voltage for Vibration Suppression in Cantilever Beam

Before applying PID controller to the cantilever beam, it is necessary to study the behavior of smart structure in vibration control. This section demonstrates the behavior of the piezoelectric elements in vibration control using FEM for passive open loop control and it is essential for applying closed loop control logic. Fig. 4 shows the finite element model upon which the active vibration control is implemented. The direction of polarization is aligned along the z-axis of the actuator. The upper and lower surfaces of the actuators are considered as the electrodes which are orthogonal to the z-axis, i.e., polarization axis. The maximum value of control

voltage for the single and double patch configuration is taken from [11] and is used for the subsequent transient analysis for voltage clipping.

V. RESULTS AND DISCUSSION

A. PID control in 3 Degree of Freedom Spring Mass System

Modal analysis of the system is performed to identify the modal frequencies followed by subsequent harmonic and transient analysis are performed to study the response characteristics. The final objective is to damp out the vibration of mass 3. Here we use a PID controller for damping out the vibration. The logic used here is to determine the error signal from the output, then feedback the error signal to PID controller and give the corrected input. The controller values can be changed to determine the damping efficiency. The PID control algorithm used for vibration control in ANSYS format is [12],

$$err = 1 - (x1 + x2 + x3)/3$$

$$sum = sum + err * dt$$

$$diff = (err - errp)/dt$$

$$u = kp * err + ki * sum + kd * diff$$

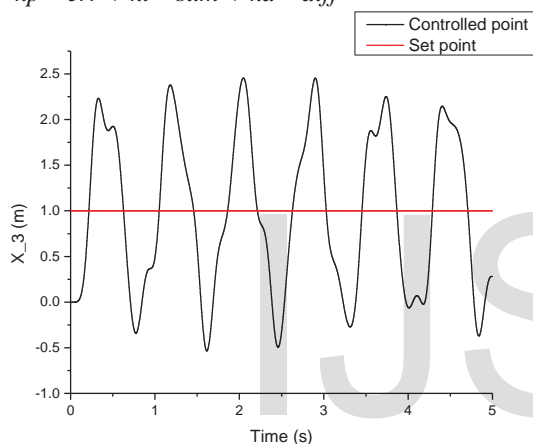


Fig (5) Uncontrolled 3 Degree of Freedom Spring Mass System

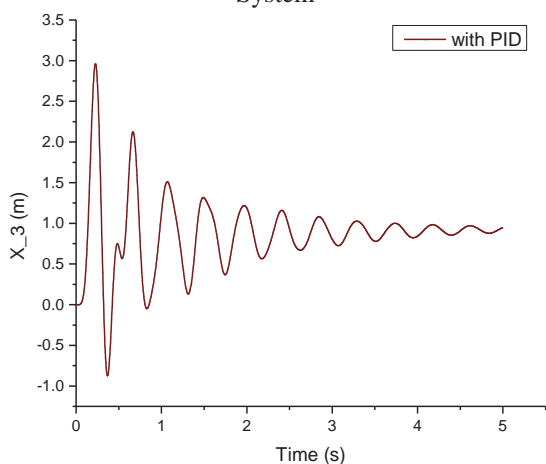


Fig (6) PID Controlled Spring Mass System

By comparing the graphs plotted in fig. 5 and fig. 6 it is quite evident that PID based control is an effect way to damp the vibration generated in a system when exposed to resonant frequencies.

B. Behavior of Piezo Patch Due to Change in Capacitance

The equation (2) that governs the capacitance in a piezoelectric patch states that the magnitude of capacitance can vary with respect to change in area “A” of piezo patch and thickness “h” between the piezo plates. This test helps to understand the way by which capacitance can make changes to damping characteristics of a smart beam. In this test the dimension of patch is changed with respect to area and then with respect to thickness.

The first case defines the behavior of capacitance when the thickness of patch is changed.

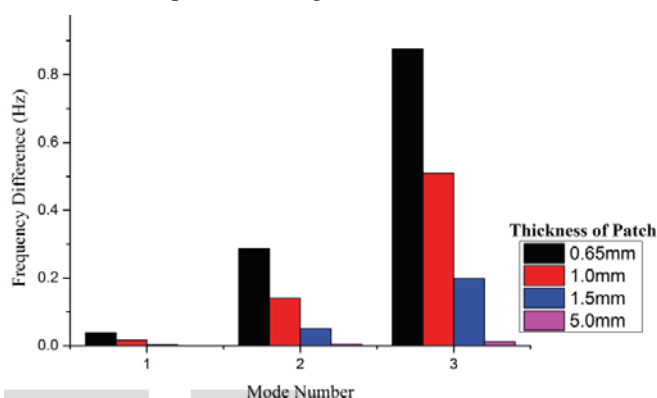


Fig (7) Change in Natural Frequency w.r.t Change in Thickness of Patch

The Frequency difference at each mode was determined by finding the difference of values in open circuit with that of short circuit. The graph represented in Fig. 7 shows that the capacitance of the piezoelectric patch decreases with the increase in the thickness of patch. The second case defines the behavior of capacitance when the area of patch is changed. In this case only the length is changed and the width is kept constant at 25mm.

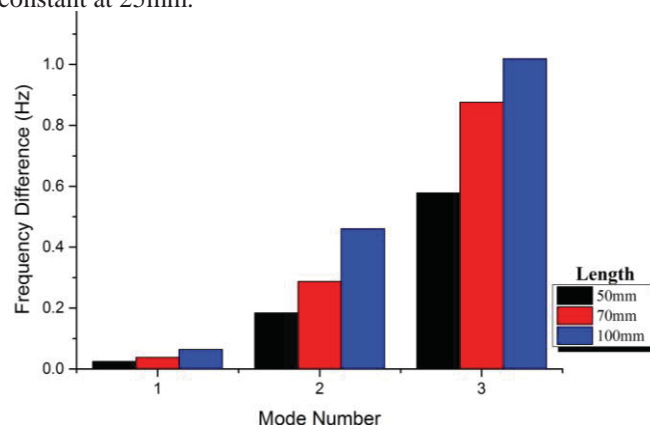


Fig (8) Change in Natural Frequency w.r.t Change in Area of Patch (Only length is changed, width is kept constant)

The graph shown in Fig. 8 shows that the property of capacitance increases with the increase in area of piezoelectric patch and the increase in capacitance offers an

increased stiffness of the piezo patch resulting in the increase in natural frequency. The short circuit nullifies the capacitor impedance and results in lower patch stiffness and hence lower frequency than the open circuit configuration.

The equation (2) shows that for a given actuator length, the capacitance increases with the square of the number of layers. Therefore, the capacitance of a piezo actuator constructed of 100 μm thick layers is 100 times the capacitance of an actuator with 1 mm layers, if the two actuators have the same dimensions. Although the actuator with thinner layers draws 100 times as much current, the power requirements of the two actuators in this example are about the same.

C. Single/double patch with PID based strain feedback controller

To show the effectiveness of double patch over single patch, a transient analysis is carried out by applying an impulsive force of 0.01N at the free end of the beam. A graph is plotted for predicting the responses with single and double patch and is shown in fig. 9. From the graph it is evident that, active vibration control of cantilever beam using PID based strain feedback closed loop control with double patch is more effective than single patch.

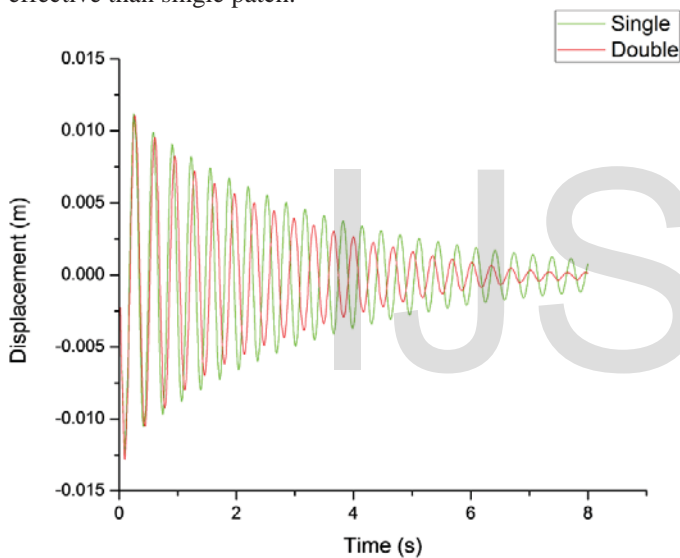


Fig (9) Comparison of vibration control using single and double patch with strain feedback PID controller

VI. CONCLUSION

Modal analysis of a three degrees of freedom spring mass system was carried out using ANSYS to identify the mode frequencies in order to compute the critical time step of the system. Three dimensional model of cantilever beam coupled with piezoelectric actuator was modelled using finite element method to compute the different types of parametric conditions on the cantilever beam.

The effect of capacitance on the behavior of stiffness in piezoelectric patch during vibration control was studied using open and short circuiting methods through modal analysis. The graphs conclude that the capacitance will increase with the

increase in area of plate and decrease with the increase in thickness between plates. The increase of plate area as well as the decrease in patch thickness lead to greater value of capacitance and hence greater charge accumulation resulting in more stiffness and will increase the natural frequency for open circuit configuration.

Vibration control for both single patch and double patch using PID based active strain feedback control mechanism was evaluated through transient analysis. It is evident that, the active vibration control using two patches is efficient and it can prevent high voltage dielectric breakdown in piezoelectric patches. PID based active vibration control with single and double patches using strain feedback control was successfully implemented. PID based active vibration control is more effective with double patch using strain feedback control.

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